Homework 3

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I.

1. Cold ⇒ Cold is valid.

|  |  |
| --- | --- |
| Cold | Cold ⇒ Cold |
| T | T |
| F | T |

1. Cold ⇒ Wet is neither.

|  |  |  |
| --- | --- | --- |
| Cold | Wet | Cold ⇒ Wet |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

1. (Cold ⇒ Wet) ⇒ (¬ Cold ⇒ ¬ Wet) is neither.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Cold | Wet | Cold ⇒ Wet | ¬ Cold ⇒ ¬ Wet | (Cold ⇒ Wet) ⇒ (¬ Cold ⇒ ¬ Wet) |
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | F |
| F | F | T | T | T |

1. Cold ∨ Wet ∨ ¬ Wet is valid.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Cold | Wet | ¬ Wet | Cold ∨ Wet | Cold ∨ Wet ∨ ¬ Wet |
| T | T | F | T | T |
| T | F | T | T | T |
| F | T | F | T | T |
| F | F | T | F | T |

1. ( (Cold ⇒ Wet) ⇒ (Wet ⇒ Cloudy)) is neither.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Cold | Wet | Cloudy | Cold ⇒ Wet | Wet ⇒ Cloudy | ((Cold ⇒ Wet) ⇒ (Wet ⇒ Cloudy)) |
| T | T | T | T | T | T |
| T | T | F | T | F | F |
| T | F | T | F | T | T |
| T | F | F | F | T | T |
| F | T | T | T | T | T |
| F | T | F | T | F | F |
| F | F | T | T | T | T |
| F | F | F | T | T | T |

1. (Cold ⇒ Cloudy) ⇒ ((Cold ∧ Wet) ⇒ Cloudy) is valid.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Cold | Wet | Cloudy | Cold ⇒ Cloudy | Cold ∧ Wet | (Cold ∧ Wet) ⇒ Cloudy | (Cold ⇒ Cloudy) ⇒ ((Cold ∧ Wet) ⇒ Cloudy) |
| T | T | T | T | T | T | T |
| T | T | F | F | T | F | T |
| T | F | T | T | F | T | T |
| T | F | F | F | F | T | T |
| F | T | T | T | F | T | T |
| F | T | F | T | F | T | T |
| F | F | T | T | F | T | T |
| F | F | F | T | F | T | T |

1. Warm ∨ Sunny ∨ (Warm⇒ Sunny) is valid.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Warm | Sunny | Warm ∨ Sunny | Warm ⇒ Sunny | Warm ∨ Sunny ∨ (Warm⇒ Sunny) |
| T | T | T | T | T |
| T | F | T | F | T |
| F | T | T | T | T |
| F | F | F | T | T |

1. (Warm ∧ Sunny) ∨ ¬ Sunny is neither.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Warm | Sunny | ¬ Sunny | Warm ∧ Sunny | (Warm ∧ Sunny) ∨ ¬ Sunny |
| T | T | F | T | T |
| T | F | T | F | T |
| F | T | F | F | F |
| F | F | T | F | T |

1. ( (Rain ⇒ Wet) ∧ (Wet ⇒ Cold) ) ⇒ (Rain ⇒ Cold) is valid.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Rain | Wet | Cold | Rain ⇒ Wet | Wet ⇒ Cold | (Rain ⇒ Wet) ∧ (Wet ⇒ Cold) | Rain ⇒ Cold | ( (Rain ⇒ Wet) ∧ (Wet ⇒ Cold) ) ⇒ (Rain ⇒ Cold) |
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | F | T | T |
| T | F | F | F | T | F | F | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | F | T | T |
| F | F | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T |

1. ( (Rain ∨ Wet ) ∧ ( ¬ Wet ∨ Cold) ) ⇒ (Rain ∨ Cold) is valid.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Rain | Wet | Cold | Rain ∨ Wet | ¬ Wet ∨ Cold | (Rain ∨ Wet ) ∧ ( ¬ Wet ∨ Cold) | Rain ∨ Cold | ( (Rain ∨ Wet ) ∧ ( ¬ Wet ∨ Cold) ) ⇒ (Rain ∨ Cold) |
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | T | T | T | T | T |
| T | F | F | T | T | T | T | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | F | F | T |
| F | F | T | F | T | F | T | T |
| F | F | F | F | T | F | F | T |

II.

1. No, it isn’t. “DogSleeps ∧ HouseWarm ∧ NightQuiet” means “the dog sleeps, and the house is warm, and the night is quiet.”
2. Yes, it is. “(DogSleeps ∧ HouseWarm) ⇒ NightQuiet” means exactly "If the dog sleeps and the house is warm, then the night is quiet."
3. No, it isn’t. “(DogSleeps ∨ HouseWarm) ⇒ NightQuiet” means “If the dog sleeps or the house is warm, then the night is quiet.”
4. No, it isn’t. “NightQuiet ⇒ (DogSleeps ∧ NightQuiet)” means “If the night is quiet, then the dog sleeps and the house is warm.”
5. No, it isn’t. “¬ DogSleeps ∨ (¬ NightQuiet ∨ HouseWarm)” means “the dog doesn’t sleep, or either the night is not quiet or the house is warm.”

III.

First, we need to convert all these propositional clauses into CNF format:

1. ( Cold ∧ Dry ) ⇒ Pleasant

**≡** ¬ ( Cold ∧ Dry ) ∨ Pleasant

**≡** ¬ Cold ∨ ¬ Dry ∨ Pleasant

1. January ⇒ ( Winter ∧ Wet )

**≡** ¬ January ∨ ( Winter ∧ Wet )

**≡** (¬ January ∧ Winter) ∨ ( ¬ January ∧ Wet )

1. Winter ⇒ Dry

**≡** ¬ Winter ∨ Dry

1. Winter ⇒ Cold

**≡** ¬ Winter ∨ Cold

1. January
2. ¬ Pleasant (rep. Goal)

Prove by resolution with refutation “Pleasant”:

1. ¬ Cold ∨ ¬ Dry ∨ Pleasant premise
2. (¬ January ∧ Winter) ∨ ( ¬ January ∧ Wet ) premise
3. ¬ Winter ∨ Dry premise
4. ¬ Winter ∨ Cold premise
5. January premise
6. ¬ Pleasant rep. Goal
7. Winter ∧ Wet 2, 5

7a. Winter 7

7b. Wet 7

1. Cold 4, 7a
2. Dry 3, 7a
3. ¬ Dry ∨ Pleasant 1, 8
4. Pleasant 10, 9
5. ⟂ 6, 11

* Combine #5 (January) with #2 (¬ January ∧ Winter) ∨ ( ¬ January ∧ Wet ) to get #7 (Winter ∧ Wet)
* Separate #7 (Winter ∧ Wet) into #7a (Winter) and #7b (Wet)
* Combine #4 (¬ Winter ∨ Cold) and #7a (Winter) to get #8 (Cold)
* Combine #3 (¬ Winter ∨ Dry) and #7a (Winter) to get #9 (Dry)
* Combine #8 (Cold) and #1 (¬ Cold ∨ ¬ Dry ∨ Pleasant) to get #10 (¬ Dry ∨ Pleasant)
* Combine #9 (Dry) and #10 (¬ Dry ∨ Pleasant) to get #11 (Pleasant)
* Combine #11 (Pleasant) and rep. Goal (¬ Pleasant) to get #12 (⟂ Contradiction)

IV.

First, we need to convert all these propositional clauses into CNF format:

1. ¬ A ⇒ ( B ∨ C )

**≡** ¬¬ A ∨ ( B ∨ C )

**≡** A ∨ B ∨ C

1. A ⇒ B

**≡** ¬ A ∨ B

1. ¬ (¬ B ⇒ D)

**≡** ¬ (B ∨ D)

**≡** ¬ B ∧ ¬ D

1. ¬ (C ∧ ¬ D) (rep. Goal)

**≡** ¬ C ∨ D

Prove by resolution with refutation “C ∧ ¬ D”:

1. A ∨ B ∨ C premise
2. ¬ A ∨ B premise
3. ¬ B ∧ ¬ D premise
4. ¬ C ∨ D premise
5. ¬ B 3
6. ¬ D 3
7. ¬ C 4, 6
8. A ∨ B 1, 7
9. A 8, 5
10. B 2, 9
11. ⟂ 5, 10

* Separate #3 (¬ B ∧ ¬ D) into #5 (¬ B) and #6 (¬ D)
* Combine #1 (A ∨ B ∨ C) and #7 (¬ C) to get #8 (A ∨ B)
* Combine #8 (A ∨ B) and #5 (¬ B) to get #9 (A)
* Combine #2 (¬ A ∨ B) and #9 (A) to get #10 (B)
* Combine #5 (¬ B) and #10 (B) to get #11 (⟂ Contradiction)